A Novel Estimate of the Positron Displacement on Image Resolution in Quantitative Nuclear Medicine

CAP 2024

Philippe Laporte

Ph.D. Student,
Université de Montréal, PQ, Canada
Under the supervision of Dr. Jean-François Carrier, MCCPM
philippe.laporte.3@umontreal.ca

May 29^{th}, 2024

Overview

Disclaimers

- Work in Progress!
- This is a first-order analysis.

Context: Nuclear Medicine

- Nuclear medicine uses radiopharmaceuticals to detect functional activity in an organism;
- It is a type of functional imaging, i.e. not anatomical;
- An anatomical image (eg. CT, MRI, US) can be added to have both anatomical and functional features;
- Many physical factors affect the quality of the produced image.

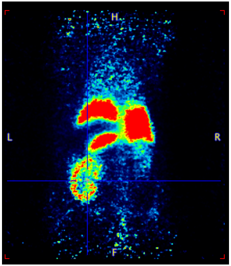


Figure: Nuclear medicine acquisition of a rat injected with a radiopharmaceutical gathering in the kidneys.

Context: Nuclear Medicine

- The displacement of the positron affects the resolution of the image;
- With smaller detector size, this becomes important;
- In clinical studies, the size is roughly 4 mm and, in preclinical studies, 0.4 mm.

Isotope	E_{mean} (keV)	E_{max} (keV)	R_{mean} (mm)	R_{max} (mm)
^{18}F	252	635	0.660	2.633
^{11}C	390	970	1.266	4.456
^{13}N	488	1190	1.730	5.572
^{15}O	730	1720	2.965	9.132

Table: Mean and maximal values for the energy and range of emitted positrons in water (Jødal, 2012)

Goal: Positron Displacement

- The current goal was to quantify the loss of resolution caused by the positron displacement;
- The aim was to use this knowledge to appreciate segmentations in a dynamic context.

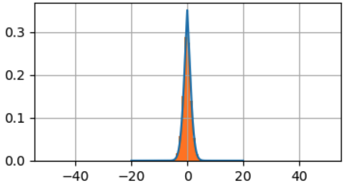


Figure: Distribution of the displacement of the positron (x axis in mm)

Assumptions

- Many assumptions are taken for granted in this presentation:
  1. Perfect detections;
  2. Angle of 180° between the annihilation photons;
  3. Straight displacement of the positron (CSDA Range);
  4. Suitable reconstruction scheme (including beam hardening and attenuation corrections).

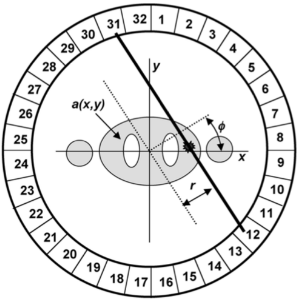


Figure: Ideal Line of Response (LOR) in a detector

Detection: Movement within a Line Voxel

- A detection within a detector can occur from a positron that originated there or not;
- If it came from within, it is good;
- If it came from without, it is bad.

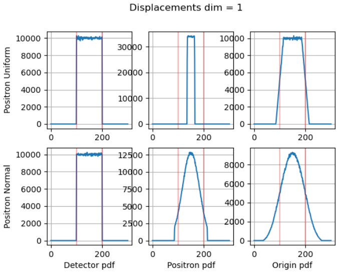


Figure: Estimation of the origin of the positron in 1D

Detection: Estimations

- Two methods were used to estimate the fraction of good detections:
  - Monte Carlo simulations;
  - Analytic model.
- The two approaches gave agreeing results.

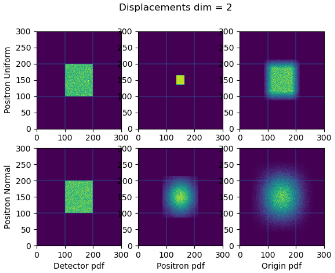


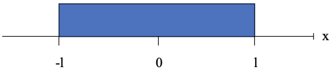
Figure: Estimation of the origin of the positron in 2D

Detection: Estimations - Analytic

- The analytic method lead to a closed formula:

$$G(l, r_{max}) = \begin{cases} \frac{1}{l}\left\{(l - r_{max}) + \int_{l-r_{max}}^{l} \int_{-r_{max}}^{l-x} \xi(r) dr dx \right\} & \text{, if } l > r_{max} \\ \frac{1}{l} \int_{0}^{l} \int_{-r_{max}}^{l-x} \xi(r) dr dx & \text{, if } l = r_{max} \\ \frac{1}{l}\left[\int_{0}^{r_{max}-l} \int_{-(l+x)}^{l-x} \xi(r) dr dx + \int_{r_{max}-l}^{l} \int_{-r_{max}}^{l-x} \xi(r) dr dx \right] & \text{, if } \frac{1}{2} r_{max} \leq l < r_{max} \\ \frac{1}{l} \int_{0}^{l} \int_{-(l+x)}^{l-x} \xi(r) dr dx & \text{, if } l < \frac{1}{2} r_{max} \end{cases}$$

$2l$ is the size of the detector, r_{max} is the maximum range of the positron and $\xi(r)$ is the pdf of the positron range.



Detection: Estimations - Analytic

- By using a uniform pdf, one can get

$$G(l, r_{max}) = \begin{cases} 1 - \frac{r_{max}}{4l} & \text{, if } l > r_{max} \\ \frac{3}{4} & \text{, if } l = r_{max} \\ \frac{r_{max} - l}{r_{max}} + \frac{4l^2 - r_{max}^2}{4l r_{max}} & \text{, if } \frac{1}{2} r_{max} \leq l < r_{max} \\ \frac{l}{r_{max}} & \text{, if } l < \frac{1}{2} r_{max} \end{cases} \tag{1}$$

- This model is useful to check the convergence and accuracy of the algorithm.

Positron Displacement: Range Distribution

- The pdf model used for the range of the positron was

$$g_{2D}(\delta) = (2A\delta + B)e^{-A\delta^2 - B\delta^2}, \quad (2)$$

where δ is the displacement from the origin and A and B are factors for the distributions.

Isotope	A (mm^{-2})	B (mm)
^{18}F	0.2951	1.5090
^{11}C	0.1262	0.7010
^{13}N	0.0885	0.4681
^{15}O	0.0436	0.2266

Table: Values for A and B used in equation 2 (Jødal, 2012)

Results: 1D

- Analytic and Monte Carlo results agree:

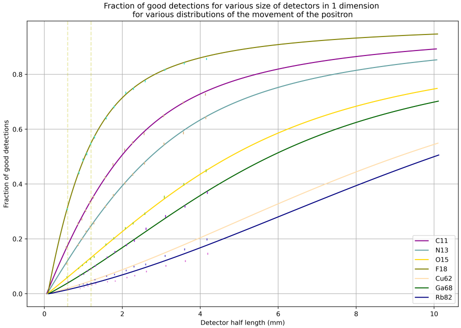


Figure: Results in 1D

Results: 3D

- For higher n-dimensional results,

$$G_n(l, r_{max}) = [G(l, r_{max})]^n$$

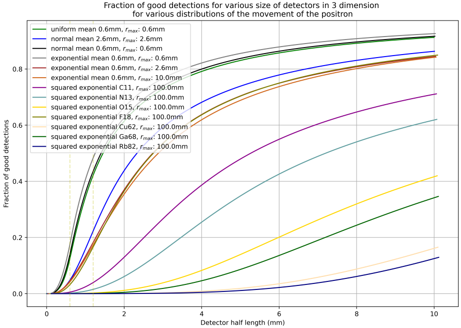


Figure: Results in 3D

Next Steps

- Use this model in the context of a strong magnetic field (PET-MRI);
- See the impact on the sinogram directly;
- Compare to the recovery coefficients used in PET.

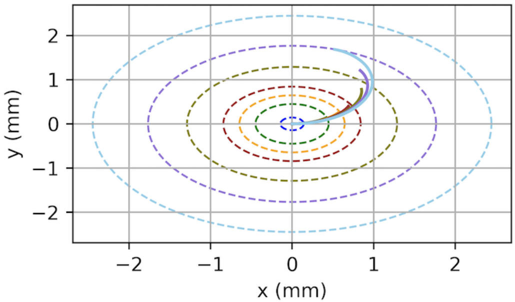


Figure: Trajectory in a magnetic field

Summary

- Positron displacement in PET can be relevant, especially with smaller detector sizes;
- A novel estimate of the loss of spatial resolution was derived analytically;
- This model uses basic assumptions, but can and shall be expanded in the future.

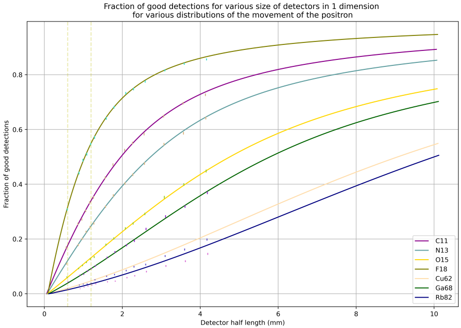


Figure: Results in 1D

References

1 Cherry, S.R. et Dahlbom, M. *PET : Physics, Instrumentation and Scanners*. Springer: Londres. 2006.

2 Jødal, L., Le Loirec, C., and Champion, C. *Positron range in PET imaging: an alternative approach for assessing and correcting the blurring*. Phys. Med. Biol. **57** (2012) 3931-43.

3 Srinivas, S. M., Dhurairaj, T., Basu, S., Bural, G., Surti, S. and Alavi, A. *A recovery coefficient method for partial volume correction of PET images.* Ann. Nucl. Med. **23** (2009) 23: 341-348.

Convergence of the Monte Carlo

- The equation of $G(l, r_{max})$ can be solved for a uniform pdf for the positron displacement;
- This allows to confirm the convergence of the simulations:

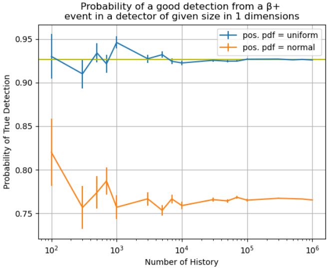


Figure: Convergence of the MC in 1D

Magnetic Field

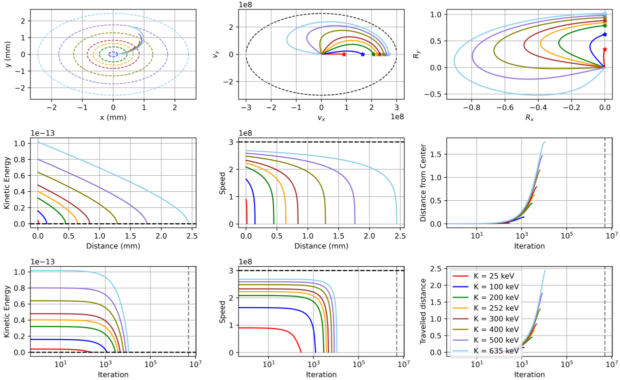


Figure: Positron movement in 2D in a magnetic field

Implementation

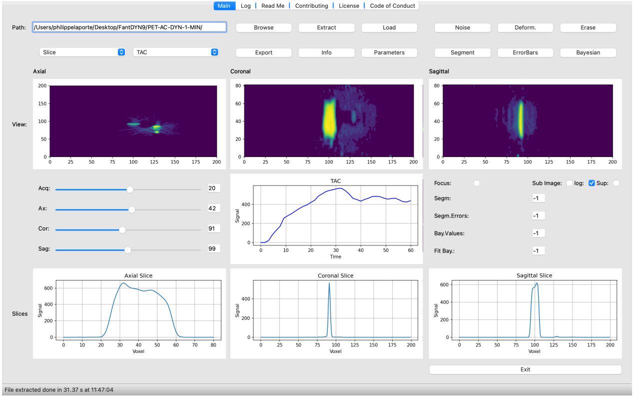


Figure: TRU-IMP GUI: Download

Recovery Coefficients

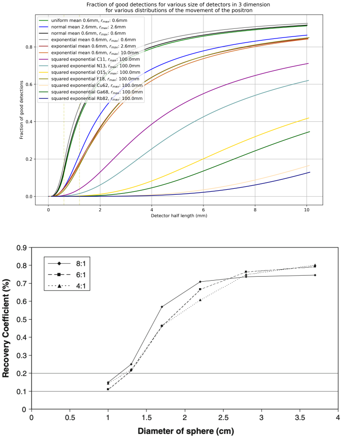


Figure: Results in 3D